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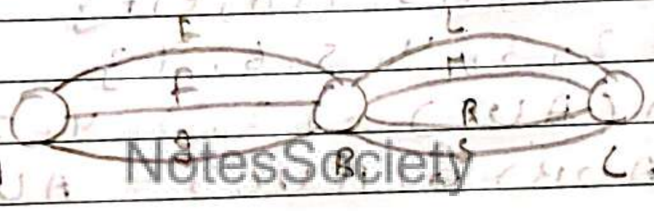
Module-3 Combinational Analysis

1) mn-theorem or rule of counting.

If one work can be done in 'm' different ways used, when it is done in any one of these ways, a second work can be done in 'n' different ways then the two works in succession can be done in 'mn' different ways.

Eg:-



In how many ways one can go from A to C via B?

→ No. of ways A can go B :- 3
No. of ways B can go C :- 4
∴ No. of ways A to C = $3 \times 4 = 12$.

11) Permutation:- The arrangement or orderly placement of things. Each of the arrangements that can be made by taking 'r' object from 'n' different objects ($r \le n$) is called a permutation.

Denotted by:- $^nP_r = \frac{n!}{(n-r)!}$

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(iii) Factorial - n (n-factorial):-
The continuous product of 'n' natural numbers (i.e. 1 to n) is called factorial n or n-factorial. It is denoted by n!

$1! = 1$ $\quad\quad 0! = 1$.
$2! = 1 \cdot 2$
$3! = 1 \cdot 2 \cdot 3$
$4! = 1 \cdot 2 \cdot 3 \cdot 4$
$n! = 1 \cdot 2 \cdot 3 \cdot 4 \ldots n$

Eg:- The result ①, show that $n! = (n-1)!\, n$
$n! = n(n-1)!$

proof:-
Since:- $n! = 1 \cdot 2 \cdot 3 \cdot 4 \ldots n-1 \cdot n$
$\Rightarrow n! = (n-1)! \cdot n$

$\Rightarrow n! = n \cdot (n-1) \cdot (n-2)!$

$\Rightarrow n! = n(n-1)(n-2)(n-3)!$

Result ②:- Show that $^nP_r = n(n-1)(n-2) \ldots (n-r-1)$

or $^nP_r = \frac{n!}{(n-r)!}$

Proof:- First place can be filled in (n-0) different ways.
Second place can be filled in (n-1) different ways.
Third place can be filled in (n-(r-1)) different ways.
∴ 'r' place can be filled in (n-(r-1)) diff. ways.

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∴ By 'mn' theorem:-

'r' place by succession = $n(n-1)(n-2)\ldots(n-r+1)$

$$\Rightarrow \frac{n(n-1)(n-2)(n-(r-1))(n-r)(n-r-1)\ldots\ldots 3\cdot 2\cdot 1!}{(n-r)(n-r-1)\ldots\ldots 3\cdot 2\cdot 1!}$$

$$\Rightarrow \frac{n!}{(n-r)!}$$

Example:-

$\rightarrow {}^{6}P_{4}$

$$\Rightarrow {}^{n}P_{r} \Rightarrow \frac{n!}{(n-r)!}$$

$$\Rightarrow \frac{6!}{(6-4)!}$$

$$\Rightarrow \frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 1}$$

$$\Rightarrow \frac{30\cdot 24}{2}$$

$$= \frac{720}{2}$$

$$P = 360$$

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$\rightarrow {}^7P_7$ $\rightarrow {}^4P_3$ $\rightarrow {}^8P_3$.

$\rightarrow {}^6P_5$.

$\rightarrow {}^7P_4$

$\rightarrow {}^5P_4$

$\rightarrow {}^5P_3$ $\rightarrow {}^7P_6$

$\rightarrow {}^9P_5$. $\rightarrow {}^9P_7$

$\Rightarrow {}^7P_7 \Rightarrow \frac{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{(7-7)!}$

$= \cancel{\text{not defined}} : 5040$.

${}^7P_4 \Rightarrow \frac{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{(7-4)!}$

$= \frac{7\cdot6\cdot5\cdot4\cdot\cancel{3\cdot2\cdot1}}{\cancel{3\cdot2\cdot1}}$

$= 840$

${}^5P_3 \rightarrow \frac{5\cdot4\cdot3\cdot\cancel{2\cdot1}}{\cancel{2\cdot1}}$

$= 60$

${}^9P_5 \Rightarrow \frac{9\cdot8\cdot7\cdot6\cdot5\cdot\cancel{4\cdot3\cdot2\cdot1}}{\cancel{4\cdot3\cdot2\cdot1}}$

$\Rightarrow 15120$

${}^4P_3 \rightarrow \frac{4\cdot3\cdot2\cdot1}{1}$

$\Rightarrow 24$

${}^6P_5 \Rightarrow \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{1}$

$\Rightarrow 30 \times 24$

$\Rightarrow 720$.

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$\rightarrow {}^5P_4$

$\Rightarrow \dfrac{5\cdot4\cdot3\cdot2\cdot1}{1}$

$\Rightarrow \cancel{120}\ \cancel{60}\ 120$

$\Rightarrow {}^7P_6$

$\Rightarrow \dfrac{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{1}$

$\Rightarrow 42 \times \cancel{120}\ 120$

$\Rightarrow 5040.$

$\rightarrow {}^9P_7$

$\Rightarrow \dfrac{9\cdot8\cdot7\cdot6\cdot5\times4\cdot3\cdot\cancel{2\cdot1}}{\cancel{2\cdot1}}$

$\Rightarrow 72\times 42\times 60$

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$\rightarrow {}^8P_3$

$\Rightarrow \dfrac{8\cdot7\cdot6\cdot5\cdot4\cdot\cancel{3\cdot2\cdot1}}{\cancel{3\cdot2\cdot1}}$

$\Rightarrow 56 \times 120$

$\Rightarrow 6720.$

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→ Since one is not to be taken everytime therefore the required no. of permutation will be said as the nine different ~~different~~ objects 4 taken at a time.

$^9P_4$

$$\Rightarrow \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}$$

$\Rightarrow 72 \times 42$

$\Rightarrow 3024$

→ In how many ~~days~~ ways, the letter of words MISSISSIPPI can be arranged in the given words. there are 11 letters can be letters of which 1111, 4 times S, 2 times P ; then required no. of permutation.

$$\rightarrow \frac{11!}{4! \times 4! \times 2!}$$

$$\Rightarrow \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{4 \cdot 3 \cdot 2 \cdot 1 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1} \cdot 2 \cdot 1}$$

$$\Rightarrow \frac{110 \times 72 \times 42 \times 5}{48}$$

$P \Rightarrow 34650$

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→ In how many ways, letter of words ACCOONTANTT, can be arranged

$\Rightarrow \frac{11!}{2!\,2!\,2!\,2!\,3!}$

$\Rightarrow \frac{11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot\cancel{4\cdot3\cdot2\cdot1}}{2\cdot1\cdot2\cdot1\cdot\cancel{2\cdot1\cdot2\cdot1}\cdot\cancel{3\cdot2\cdot1}}$

$= \frac{110\times\cancel{72}^{18}\times42\times5}{\cancel{4}}$

$= 110\times18\times42\times5$

$= 550\times18\times42$

$= 415800.$

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Combination

Each of the collection obtained by taking r (r ≤ n), objects from and different objects is known as combination.

The symbol ⇒ ^nC_r is used to denote the total no. of combination obtained by taking are r. Object from n, different object in each time.

And different ~~determinative~~ iterative system Symbol for denoting combination :- $\binom{n}{r}$

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As for eg:- $^3C_2$, sense the total no. of combinations, obtain by taking two objects ~~from~~ 3 diff. 2.

Theorem:-

⇒ The no. of combinat n diff. object taken are r at a. ie, ^nC_r

given by :- $^nC_r = \frac{n!}{r!\,(n-r)!}$.

→ $^6C_2$

→ $^9C_3$

→ $^5C_4$

→ $^7C_2$

→ $^5C_3$

→ $^9C_4$

→ $^{10}C_5$

→ $^4C_2$

→ $^4C_4$

→ $^{12}C_6$

$$^6C_2 = \frac{6!}{2!\,(4)!}$$

$$= \frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2! \times 4\times 3\times 2\times 2\times 1}$$

$$= \frac{30}{2} = 15$$

$\Rightarrow {}^9C_3$

$$\Rightarrow \frac{9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{3!\times 6!}$$

$$= \frac{72\times 7}{3\times 2\times 1}$$

$$= 12\times 7$$

$$= 84.$$

$\Rightarrow {}^5C_4$

$$\Rightarrow \frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{4!\times 1!}$$

$$= 5.$$

$\Rightarrow {}^7C_2$

$$\Rightarrow \frac{7\cdot \overset{3}{6}\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2!\times 5!}$$

$$= 21$$

$\Rightarrow {}^5C_3$

$$\Rightarrow \frac{5\cdot \overset{2}{4}\cdot 3\cdot 2\cdot 1}{3!\times 2!}$$

$$\Rightarrow 10.$$

$\Rightarrow {}^{9}C_{4}$

$$\Rightarrow \frac{9 \times 8 \times 7 \times 6 \times \cancel{5 \times 4 \times 3 \times 2 \times 1}}{4! \times \cancel{5!}}$$

$$\Rightarrow \frac{72 \times 42}{4 \times 3 \times 2 \times 1}$$

$$= \frac{72 \times 42}{24}$$

$$= 126.$$

$\Rightarrow {}^{10}C_{5}$

$$\Rightarrow \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{5! \times \cancel{5!}}$$

$$\Rightarrow \frac{\overset{18}{\cancel{90}} \times \overset{14}{\cancel{56}} \times \cancel{6}}{\cancel{5} \times \cancel{4} \times \cancel{3 \times 2 \times 1}}$$

$$\Rightarrow 252.$$

$\Rightarrow {}^{4}C_{2}$

$$\Rightarrow \frac{\overset{2}{\cancel{4}} \times 3 \times \cancel{2 \times 1}}{\cancel{2!} \times \cancel{2!}}$$

$$\Rightarrow 6.$$

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$\Rightarrow {}^{11}C_4$

$$\Rightarrow \frac{11 \times 10 \times 9 \times 8 \times \cancel{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}}{4! \times \cancel{7!}}$$

$$= \frac{110 \times 72}{24}$$

$$= 330.$$

$\Rightarrow {}^{12}C_6$

$$\Rightarrow \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times \cancel{6 \times 5 \times 4 \times 3 \times 2 \times 1}}{6! \times \cancel{6!}}$$

$$\Rightarrow \frac{12 \times 110 \times 72 \times 7}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= \frac{12 \times 110 \times 72 \times 7}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= 924.$$

→ Out of 8 persons, the team of 5 can be selected, in how many ways?

→ Out of 7 boys & 4 girls, a team of 8 can selected in how many ways?

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$\rightarrow {}^{8}C_{5}$

$$\Rightarrow \frac{8\times7\times6\times\cancel{5\times4\times3\times2\times1}}{\cancel{5!}\times3!}$$

$$\Rightarrow \frac{56\times\cancel{6}}{\cancel{3\times2\times1}}$$

$\Rightarrow 56$

$\rightarrow {}^{11}C_{8}$

$$\Rightarrow \frac{11\times10\times9\times\cancel{8\times7\times6\times5\times4\times3\times2\times1}}{\cancel{8!}\times3!}$$

$$\Rightarrow \frac{\overset{55}{\cancel{110}}\times\overset{3}{\cancel{9}}}{\cancel{3}\times\cancel{2}\times1}$$

$= 165$

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→ Find the number of permutations of the letters in the word ALLAHABAD.

→ ^nP_r

$$\Rightarrow \frac{9!}{4! \times 2!}$$

$$\Rightarrow \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 2 \times 1}$$

$$\Rightarrow 72 \times 15 \times 7$$

$$\Rightarrow 72 \times 105 \Rightarrow 7560$$

→ A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man & 2 women?

$$\Rightarrow {}^nP_r = \frac{5!}{3!(5-3)!}$$

$$\Rightarrow \frac{5!}{3! \times 2!}$$

$$= \frac{5 \times 4 \times 3 \times 2 \times 1}{3! \; 2!}$$

$$\Rightarrow \frac{20}{2} = \boxed{10}$$

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Committees with 1 man & 2 women:-

→ Choosing 1 man from 2 ⇒ $\binom{2}{1} = 2$.

→ Choosing 2 women from 3 ⇒ $\binom{3}{2} = 3$.

∴ According to m×n formula:-

$$\boxed{2 \times 3 = 6}$$

UNIT-4.

→ Basic Probability Concepts.

• Random experiment:-

An activity that produces n outcomes is referred to as an experiment. An experiment is said to be a random experiment, if its outcome cannot be predicted certainity.

Example:- If a coin is tossed, one cannot predict beforehand whether head or tail, will appear. So its a random experiment.

• Sample Space:-
The set of all possible outcomes of an experiment is called as the sample space. It is denoted by 'S' and its number of elements denotted by n(S).

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Example :-
While rolling a die, the number that would appear would be any one of the numbers drawn → 1, 2, 3, 4, 5, 6. so here, S = {1, 2, 3, 4, 5, 6} and n{S} = 6.
Similarly, in the case of tossing an unbiased coin S = {Head, Tail} or {T, H} and n(S) = 2.
The elements of the sample space are called sample points.

• Event :-

Every subset of a sample is an event. It is denoted by 'E'.
Example :- In throwing a dice, S = {1, 2, 3, 4, 5, 6} the appearance of an even number will be the event E = {2, 4, 6}. E is a subset of S.

• Types of Event :-

① Simple Event :-
An event, consisting of single sample point is called a sample event.
Example :- In throwing a dice, S = {1, 2, 3, 4, 5, 6}, each of {1}, {2}, {3}, {4}, {5} & {6} are simple events.

② Compound Event :-
A subset of sample space, which has more than one element is called compound event.

Ex:- In throwing a dice, the event of appearance of odd numbers is a compound event, since, $E=\{1, 3, 5\}$ has 3 elem

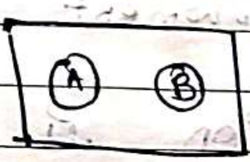
③ Equally likely events:-
Events are said to be equally likely, if there is no reason to believe that one more probable to occur than the other.
Example:- When a dice is thrown, all the six faces $\{1, 2, 3, 4, 5, 6\}$ are equally likely to come up.

④ Exhaustive events:-
Two or more events are said to be exhaustive, if they collectively constitute the sample space.
Example:- A dice is thrown, the events "an odd score" and "an even score" form an exhaustive set of events.

⑤ Sure event:-
Let 'S' be a sample space. If E equal ~~part~~ to S then E is called as sure event.
Example:- In a thrown dice,
$S = \{1, 2, 3, 4, 5, 6\}$
Let E_1 = Event of getting a no. less than 7.
$\therefore$ E_1 is a sure event.
$\therefore$ We can say, in a sure event,
$n(E) = n(S)$.

⑥ Mutually exclusive or disjoint events:-

If two or more events cannot occur simultaneously, i.e. no. of them can occur simultaneously, they are called as mutually exclusive or disjoint events.
Example:- When a coin is tossed, the event of occurance of a head and a tail are mutually exclusive events.



$A \cap B = \phi$.

⑦ Independent or mutually independent events:-

Two or more events are said to be independent, if the occurance or non occurance of any one of them doesn't affect the probability of occurance or non-occurance of other events.
Ex:- When a coin is tossed twice, the event of occurance of head in the first throw and event of occurance in the second throw are independent events.

⑧ Complement of an event:-

Let S be the sample for random experiment, an E be an event. Then complement

E is denoted by E'. Here E' occurs if and only if E doesn't occur.

- Classical definition of probability:-

If 'S' be the sample space, then the probability of occurance of an event 'E' is defined as:-

$$P(E) = \frac{n[E]}{n[S]} = \frac{\text{no. of element in } E}{\text{no. of element in } S}$$

Note:- This definition is not true, if:
a) The events are not equally likely
b) The possible outcomes are infinite.

- Properties of probability:-

1. The probability of an event lies betn '0' and '1', i.e, $0 \leq P(E) \leq 1$.

2. The probability of an impossible event is '0' i.e, $P(\emptyset) = 0$.

3. The probability of a sure event is 1. i.e, $P(S) = 1$, where 'S' is the sure event.

4. If two events 'A' & 'B' are such that $A \subset B$, then $P(A) \leq P(B)$.

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5) If 'E' is any event and E' be the complement of event E', then the P(E) + P(E') = 1.

Ques Three coins are tossed, what is the probability of getting
1) All heads
2) only 2 heads
3) Atleast one head
4) Atleast two heads.

Sample space = {HHH, TTT, HHT, HTT, HTH, THH, TTH, THT}

$n(S) = 8$.

i) All heads:- $\frac{n(E)}{n(S)} = \frac{1}{8}$

ii) Two heads = $\frac{n(E)}{n(S)} = \frac{3}{8}$

iii) Atleast one head = $\frac{7}{8}$

iv) Atleast two heads = $\frac{4}{8} = \frac{1}{2}$.

Ques. A fair coin is tossed and a fair dice is thrown.
Write down S.S for :-
→ The toss of the coin
→ The throw of the dice.

i) Toss :- $S = \{H, T\}$; $n(S) = 2$
dice :- $S = \{1, 2, 3, 4, 5, 6\}$; $n(S) = 6$

→ Probability :-

Overview :-

Conditional Probability :-

If E & F are two events associated with same sample space of a random experiment, then conditional probability of the event E under the condition that the Event F has occured written as $P(E|F)$, is given by :-

$$P(E|F) = \frac{P(F \cap E)}{P(F)}$$

$$P(F) \neq 0$$

→ Properties of conditional Probability :-

Let E & F be events associated with S of an experiment. Then :-

1) $P(S|F) = P(F|F) = 1$

11) $P[(A \cup B)|F] = P(A|F) + P(B|F) - P[(A \cap B|F)]$,
where A & B are any two events associated with S.

111) $P(E'|F) = 1 - P(E|F)$

→ Multiplication Theorem on Probability :-

Let E & F be two events associated with S of an experiment. Then,

$P(E \cap F) = P(E) P(F|E), P(E) \neq 0$
$= P(F) P(E|F), P(F) \neq 0$

If E, F, G are 3 events associated with S, then :-
$P(E \cap F \cap G) = P(E) P(F|E) P(G|E \cap F)$

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→ Independent Events :-

Let E & F be two events, associated with S. If probability of occurance of one of them is not affected by occurance of other, then we say that the two events are independent. Thus, the two Events E & F will be independent if,

a) $P(F|E) = P(F)$, provided, $P(E) \neq 0$.
b) $P(E|F) = P(E)$, provided, $P(F) \neq 0$

Using multiplication theorem, we have:

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c) $P(E \cap F) = P(E) P(F)$

Three events A, B & C are said to be mutually independent if all the following conditions hold:-

$$P(A \cap B) = P(A) P(B)$$
$$P(A \cap C) = P(A) P(C)$$
$$P(B \cap C) = P(B) P(C)$$
$$P(A \cap B \cap C) = P(A) P(B) P(C)$$

→ Partition of a sample space :-

A set of events $E_1, E_2 \ldots E_n$ is said to be represent a partition if :-

a) $E_i \cap E_j = \phi$, $i \neq j$; $i, j = 1, 2, 3 \ldots n$.

b) $E_1 \cup E_2 \cup \dots \cup E_n = S$, &

c) Each $E_i \neq \phi$, i.e., $P(E_i) > 0$ for all $i = 1, 2, \dots n$.

→ **Theorem of total probability :-**

Let $(E_1, E_2, \dots E_n)$ be a partition of S. Let A be any event associated with S, then :-

$$P(A) = \sum_{i=1}^{n} P(E_i) P(A \mid E_i)$$

→ **Bayes theorem :-**

If $E_1, E_2, \dots E_n$ are mutually exclusive & exhaustive events associated with S & A is any event of non-zero probability, then

$$P(E_i \mid A) = \frac{P(E_i) P(A \mid E_i)}{\sum_{i=1}^{n} P(E_i) P(A \mid E_i)}$$

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Complete

Unit - 5

Probability Distribution

5.1 : Probability density functions
5.2 : Cumulative distribution functions
5.3 : Expectation and Variance
5.4 : Uniform and normal distributions
5.5 : Joint probability mass and density func
5.6 : Marginal and conditional distributions
5.7 : Covariance and correlation